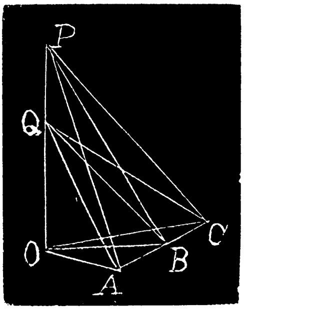
$\angle OAP=45°=\beta$, $\angle OAQ=40°=\delta$, $\angle OBP=42°=\gamma$, $\angle OBQ=38°=\mu$, $\angle OCP=40°=\lambda$, $\angle OCQ=36°=\nu$.

$OA=m$, $OB=n$, $OC=p$, $\angle OCA=\theta$.

$\therefore$ $m=y\cot\delta=x\cot\beta$, $n=y\cot\mu=x\cot\gamma$.

$p=y\cot\nu=x\cot\lambda$.

Also from triangles OCA and OCB,



$m^2=p^2+(a+b)^2-2p(a+b)\cos\theta$ ....(1).

$n^2=p^2+b^2-2pb\cos\theta$ ....(2).

The values of m, n, p in (1) and (2) and eliminating $\cos\theta$, we get

$$x=\sqrt{\frac{ab(a+b)}{b\cot^2\beta+a\cot^2\lambda-(a+b)\cot^2\gamma}} \qquad y=\sqrt{\frac{ab(a+b)}{b\cot^2\delta+a\cot^2\nu-(a+b)\cot^2\mu}}$$

Substituting we get $y=1505.183$ feet, $x=4232.505$ feet.

Also solved by *ALOIS F. KAVORIK*, and *CHAS. C. CROSS*.

CALCULUS.

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71. Proposed by J. C. CORBIN, Pine Bluff, Ark.

Form the differential equation of the third order, of which $y=c_1e^{2x}+c_2e^{-3x}+c_3e^x$ is the complete primitive.

III. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; C. W. M. BLACK, A.M., Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.; G. B. M. ZERR, A. M., Ph. D., The Russell College, Lebanon, Va.; and the PROPOSER.

$y=c_1e^{2x}+c_2e^{-3x}+c_3e^x$ .....(1), $dy/dx=2c_1e^{2x}-3c_2e^{-3x}+c_3e^x$........(2);

$d^2y/dx^2=4c_1e^{2x}+9c_2e^{-3x}+c_3e^x$....(3). $d^3y/dx^3=8c_1e^{2x}-27c_2e^{-3x}+c_3e^x$....(4).

(2)−(1) gives $dy/dx-y=c_1e^{2x}-4c_2e^{-3x}$ ....(5),

(4)−(1) gives $d^3y/dx^3-y=7c_1e^{2x}-28c_2e^{-3x}$ ....(6),

(6)−7(5) gives $d^3y/dx^3-7(dy/dx)+6y=0$.

IV. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

The equation reduces itself to what is the equation the three roots of which are 1, 2, −3. This equation is $z^2-7z+6=0$; consequently the complete primitive is $d^3y/dx^3-7(dy/dx)+6y=0$.

72. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

A man has a park in the form of a parabolic segment cut off by a chord making an angle $\frac{1}{4}\pi$ with the axis. Within the park is a right angled triangular flower plat with one vertex at the center of gravity of the segment, the other vertex at the lower extremity of the chord, and the right angle on the diameter bisecting the chord. The park contains 30 acres, and the perimeter of triangle in linear measure equals the area in square measure. Find the length of the chord, the latus-rectum of the parabola, and the dimensions of the triangle.

Solution by the PROPOSER.

Let $PQ=2y$, $BC=x$, $\angle EBC=\theta=\frac14\pi$. $\therefore y^2=4ax/\sin^2\theta=8ax$.

$\therefore$ Area$=2\sin\theta\int_o^x ydx=4\sqrt{a}\int_o^x x^{\frac12}dx=(\frac83 a^{\frac12}x^{\frac32}=\frac13(2xy\sqrt{2})$.

$\therefore \frac13(2\sqrt{2xy})=4800$ square rods, or $xy=3600\sqrt{2}$ ....(1).

If G be the center of gravity, then $BG=3x/5$.

$PD=\frac12 y\sqrt{2}$, $GD=GC-DC=\frac25x-\frac12y\sqrt{2}=\frac1{10}(4x-5y\sqrt{2})$.

$PG=\sqrt{(PD^2+GD^2)}=\frac1{10}\sqrt{(100y^2+16x^2-40xy\sqrt{2})}$.

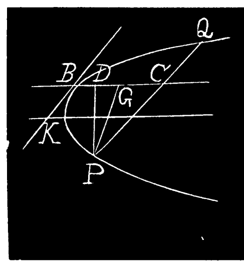
$\therefore PG=\frac1{10}\sqrt{(100y^2+16x^2-2800)}$,

$\frac12 PD.DG=\frac1{40}(4xy\sqrt{2}-10y^2)\frac14(2880-y^2)$.

$\therefore \frac1{10}\sqrt{(100y^2+16x^2-2800)}+\frac35x=\frac14(2880-y^2)$.

$\therefore 2\sqrt{(100y^2+16x^2-2800)}=14400-5y^2-8x$.

$\therefore 25y^4-144400y^2+80xy^2-230400x$
$+208512000=0$..............(2).



(1) in (2) gives

$y^5-5776y^3+11520\sqrt{2}y^2+8340480y=33177600\sqrt{2}$,

$y^5=5776y^3+16291.740672y^2+8340480y=46920213.13536$.

$\therefore y=5.6903$ *nearly*, $2y=11.3806$, length of chord.

$x=894.7101$, $PD=4.0236$, $DG=353.3604$, $PG=353.8833$, area $PDG=711.7673$, $4a=.0181=$latus rectum.

Solved with different results by *C. W. M. BLACK*, and *J. SCHEFFER*.

NOTE on solution of Problem 69, Calculus, April number, page 111: "It seems to me that this solution does not solve this question. The fence prevents the horse from grazing on the ground within it; then, the rope must extend from one end of the major axis around, *outside of the fence*, to the other end, and is twice as long as that half of the fence. Hence the horse may graze around to the end of the minor axis on the other side of the field. The horse, starting from there and keeping the rope tight, will describe a curve as the rope unwinds from the fence, until he arrives at a point opposite the other end of the minor axis, being then half way around: proceeding, he reaches the other end of the minor axis (his starting point) and describes the other half of the curve. *Josiah H. Drummond*.

MECHANICS.

64. Proposed by FREDERIC R. HONEY, Ph. B., Instructor in Trinity College, New Haven, Conn.

Let the isosceles triangle abc, whose plane is vertical, and whose base bc is horizontal, and supported at each end b and c, represent three rods jointed at the points a, b, and c. Let any load L be suspended at the vertex a. It is required to find the value of the angle between the sides of the triangle and the base which shall make the sum of the weights of the rods a minimum, the length of the base bc being fixed.